Exercise 11.1

1. (a) $a = 0$, $b = 4$, and $f(x) = x^2 - 2x$, so,

$\Delta x = \frac{4-0}{n} = \frac{4}{n}$ and $x_i = 0 + \frac{4i}{n} = \frac{4i}{n}$, and,

$$\int_0^4 (x^2 - 2x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\frac{4}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{4i}{n}\right)^2 - 2\left(\frac{4i}{n}\right)\right]\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{64i^2}{n^3} - \frac{32i}{n^2}\right] = \lim_{n\to\infty} \left[\frac{64}{n^3}\sum_{i=1}^{n} i^2 - \frac{32}{n^2}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty} \left[\left(\frac{64}{n^3}\right)\frac{n(n+1)(2n+1)}{6} - \left(\frac{32}{n^2}\right)\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[\frac{64}{6}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) - \frac{32}{2}\left(1 + \frac{1}{n}\right)\right]$$

$$= \frac{64}{6}(1)(2) - \frac{32}{2}(1) = \frac{128}{6} - \frac{32}{2} = \frac{16}{3}$$

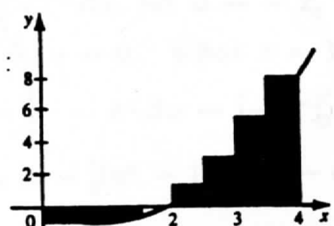
(b) $\Delta x = \frac{4-0}{8} = \frac{1}{2}$. Therefore,

$$\sum_{i=1}^{8} f(x_i)\,\Delta x = \frac{1}{2}\sum_{i=1}^{8} f(x_i)$$

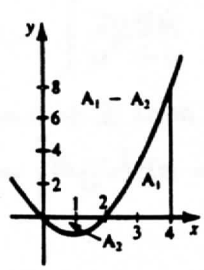
$$= \frac{1}{2}\left[f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4)\right]$$

$$= \frac{1}{2}\left[-0.75 - 1 - 0.75 + 0 + 1.25 + 3 + 5.25 + 8\right] = 7.5$$

(c)



(d) Since $f(x) = x^2 - 2x \leq 0$ for $0 \leq x \leq 2$, and $f(x) \geq 0$ for $x \geq 2$, the integral can be interpreted as $A_1 - A_2$, where A_1 and A_2 are the areas shown in the diagram.



2. (a) $a = -2$ and $b = 3$, so,

$\Delta x = \dfrac{3-(-2)}{n} = \frac{5}{n}$ and $x_i = -2 + \frac{5i}{n}$, and,

$$\int_{-2}^{3}(1-4x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-2+\tfrac{5i}{n}\right)\tfrac{5}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left[1-4\left(-2+\tfrac{5i}{n}\right)\right]\tfrac{5}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(9-\tfrac{20i}{n}\right)\tfrac{5}{n} = \lim_{n\to\infty}\left(\tfrac{45}{n}\sum_{i=1}^{n} - \tfrac{100}{n^2}\sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty}\left[\tfrac{45}{n}n - \left(\tfrac{100}{n^2}\right)\tfrac{n(n+1)}{2}\right] = \lim_{n\to\infty}\left[45 - 50(1)\left(1+\tfrac{1}{n}\right)\right]$$

$$= 45 - 50 = -5$$

(b) $a = 0$ and $b = 1$, so,

$\Delta x = \dfrac{1-0}{n} = \frac{1}{n}$ and $x_i = 0 + \frac{i}{n} = \frac{i}{n}$, and,

$$\int_{0}^{1}(1+4x-6x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\tfrac{i}{n}\right)\tfrac{1}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left[1+4\left(\tfrac{i}{n}\right)-6\left(\tfrac{i}{n}\right)^2\right]\tfrac{1}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\tfrac{1}{n}+\tfrac{4i}{n^2}-\tfrac{6i^2}{n^3}\right) = \lim_{n\to\infty}\left(\tfrac{1}{n}\sum_{i=1}^{n} + \tfrac{4}{n^2}\sum_{i=1}^{n} i - \tfrac{6}{n^3}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty}\left[\tfrac{1}{n}n + \left(\tfrac{4}{n^2}\right)\tfrac{n(n+1)}{2} - \left(\tfrac{6}{n^3}\right)\tfrac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[1 + 2(1)\left(1+\tfrac{1}{n}\right) - 1\left(1+\tfrac{1}{n}\right)\left(2+\tfrac{1}{n}\right)\right] = 1 + 2 - 2 = 1$$

(c) $a = 0$ and $b = 1$, so,

$\Delta x = \dfrac{1-0}{n} = \frac{1}{n}$ and $x_i = 0 + \frac{i}{n} = \frac{i}{n}$, and,

$$\int_{0}^{1}x^3\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\tfrac{i}{n}\right)\tfrac{1}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\tfrac{i}{n}\right)^3\tfrac{1}{n} = \lim_{n\to\infty}\tfrac{1}{n^4}\sum_{i=1}^{n} i^3$$

$$= \lim_{n\to\infty}\left(\tfrac{1}{n^4}\right)\tfrac{n^2(n+1)^2}{4} = \lim_{n\to\infty}\tfrac{1}{4}(1)\left(1+\tfrac{1}{n}\right)^2 = \tfrac{1}{4}$$

(d) $a = 1$ and $b = 4$, so,

$\Delta x = \dfrac{4-1}{n} = \frac{3}{n}$ and $x_i = 1 + \frac{3i}{n}$, and,

$$\int_{1}^{4}(x^2-6)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(1+\tfrac{3i}{n}\right)\tfrac{3}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(1+\tfrac{3i}{n}\right)^2-6\right]\tfrac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\tfrac{6i}{n}+\tfrac{9i^2}{n^2}-6\right)\tfrac{3}{n} = \lim_{n\to\infty}\left(-\tfrac{15}{n}\sum_{i=1}^{n} + \tfrac{18}{n^2}\sum_{i=1}^{n} i + \tfrac{27}{n^3}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to\infty}\left[-\tfrac{15}{n}n + \left(\tfrac{18}{n^2}\right)\tfrac{n(n+1)}{2} + \left(\tfrac{27}{n^3}\right)\tfrac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[-15 + 9(1)\left(1+\frac{1}{n}\right) + \frac{27}{6}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = -15 + 9 + \frac{27}{6}(2) = 3$$

3. (a) $a = 0$ and $b = 3$, so,

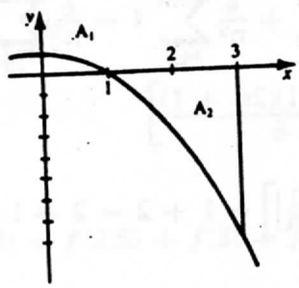
$x = \frac{3}{n}$ and $x_i = \frac{3i}{n}$, and,

$$\int_0^3 (1 - x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left[1 - \left(\frac{3i}{n}\right)^2\right]\frac{3}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{3}{n} - \frac{27i^2}{n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\sum_{i=1}^{n} 1 - \frac{27}{n^3}\sum_{i=1}^{n} i^2\right) = \lim_{n\to\infty}\left[\frac{3}{n}n - \left(\frac{27}{n^3}\right)\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[3 - \frac{27}{6}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 3 - \frac{27}{6}(2) = -6$$

Since $1 - x^2 \geq 0$ for $0 \leq x \leq 1$, and $1 - x^2 \leq 0$ for $x \geq 1$, the integral $\int_0^3 (1 - x^2)\,dx$ can be interpreted as the difference of areas $A_1 - A_2$, where A_1 and A_2 are shown in the diagram.



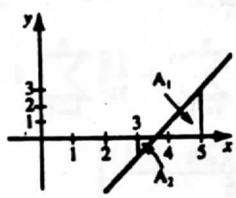
(b) $a = 3$ and $b = 5$, so,

$\Delta x = \frac{5-3}{n} = \frac{2}{n}$ and $x_i = 3 + \frac{2i}{n}$, and,

$$\int_3^5 (2x - 7)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[2\left(3+\frac{2i}{n}\right) - 7\right]\frac{2}{n} = \lim_{n\to\infty}\left(-\frac{2}{n}\sum_{i=1}^{n} + \frac{8}{n^2}\sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty}\left[-\frac{2}{n}n + \left(\frac{8}{n^2}\right)\frac{n(n+1)}{2}\right] = \lim_{n\to\infty}\left[-2 + 4(1)\left(1+\frac{1}{n}\right)\right] = -2 + 4 = 2$$

Since $2x - 7 \geq 0$ for $x \geq \frac{7}{2}$, and $2x - 7 \leq 0$ for $x \leq \frac{7}{2}$, the integral $\int_3^5 (2x - 7)\,dx$ can be interpreted as the difference of areas $A_1 - A_2$, where A_1 and A_2 are shown in the diagram.



4. $\Delta x = \frac{b-a}{n}$ and $x_i = a + \left(\frac{b-a}{n}\right)i$, and,

$$\int_a^b x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left[a + \left(\frac{b-a}{n}\right)i\right]\left(\frac{b-a}{n}\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\left[a + \left(\frac{b-a}{n}\right)i\right]^2\left(\frac{b-a}{n}\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[a^2 + 2a\left(\frac{b-a}{n}\right)i + \left[\left(\frac{b-a}{n}\right)i\right]^2\right]\left(\frac{b-a}{n}\right)$$

$$= \lim_{n\to\infty}\left[a^2\left(\frac{b-a}{n}\right)\sum_{i=1}^{n} + 2a\left(\frac{b-a}{n}\right)^2\sum_{i=1}^{n} i + \left(\frac{b-a}{n}\right)^3\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty}\left[a^2\left(\frac{b-a}{n}\right)n + 2a\left(\frac{b-a}{n}\right)^2\frac{n(n+1)}{2} + \left(\frac{b-a}{n}\right)^3\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[a^2(b-a) + a(b-a)^2(1)\left(1+\frac{1}{n}\right) + \frac{(b-a)^3}{6}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]$$

$$= a^2(b-a) + a(b-a)^2 + \tfrac{1}{3}(b-a)^3 = \tfrac{1}{3}(b^3 - a^3)$$

5. (a) Using the method developed in Chapter 5 to sketch $f(x) = x^3 - 4x$, we have for headings A – H:

A. The domain is R.

B. The y-intercept is $f(0) = 0$. The x-intercepts occur when $y = 0$, so they are $0, \pm 2$.

C. $f(-x) = -x^3 + 4x = -f(x)$, therefore, $f(x) = x^3 - 4x$ is an odd function. The curve is symmetric about the origin.

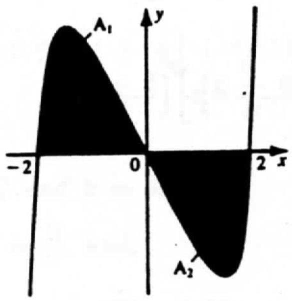
D. $\lim\limits_{n\to\infty} x^3 - 4x = \infty$ and $\lim\limits_{n\to-\infty} x^3 - 4x = -\infty$, so, there are no horizontal asymptotes. The denominator of $x^3 - 4x$ is 1, so there are no vertical asymptotes.

E. $f'(x) = 3x^2 - 4$, so, $f'(x) > 0$ when $x > \frac{2}{\sqrt{3}}$ or $x < -\frac{2}{\sqrt{3}}$ and $f'(x) < 0$ when $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$ Therefore, f is increasing on $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}}, \infty\right)$ and decreasing on $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$.

F. $f'(x) = 0$ for $x = \pm\frac{2}{\sqrt{3}}$ Therefore, by the First Derivative Test, $f\left(-\frac{2}{\sqrt{3}}\right) = \frac{16\sqrt{3}}{9}$ is a local maximum, and $f\left(\frac{2}{\sqrt{3}}\right) = -\frac{16\sqrt{3}}{9}$ is a local minimum.

G. $f''(x) = 6x \Rightarrow$ $f''(x) > 0$ for $x > 0$ and $f''(x) < 0$ for $x < 0$, so f is concave upward on $(0, \infty)$, and concave downward on $(-\infty, 0)$. Thus $(0,0)$ is the point of inflection.

H.



(b) $a = -2$, and $b = 2$, so,

$\Delta x = \frac{4}{n}$ and $x_i = -2 + \frac{4i}{n}$, and,

$$\int_{-2}^{2} (x^3 - 4x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-2 + \frac{4i}{n}\right)\left(\frac{4}{n}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-2 + \frac{4i}{n}\right)^3 - 4\left(-2 + \frac{4i}{n}\right)\right]\left(\frac{4}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(-8 + \frac{48i}{n} - \frac{96i^2}{n^2} + \frac{64i^3}{n^3} + 8 - \frac{16i}{n}\right)\left(\frac{4}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{256}{n^4} \sum_{i=1}^{n} i^3 - \frac{384}{n^3} \sum_{i=1}^{n} i^2 + \frac{128}{n^2} \sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \left[\left(\frac{256}{n^4}\right)\frac{n^2(n+1)^2}{4} - \left(\frac{384}{n^3}\right)\frac{n(n+1)(2n+1)}{6} + \left(\frac{128}{n^2}\right)\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[64(1)\left(1 + \frac{1}{n}\right)^2 - 64(1)\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 64(1)\left(1 + \frac{1}{n}\right)\right] = 64 - 64(2) + 64 = 0$$

(c) $f(x) = x^3 - 4x$ is an odd function, and therefore the symmetry of the curve about the origin demands that $A_1 = A_2$. The integral can be represented by the difference of areas $A_1 - A_2 = 0$. In fact, $\int_{-a}^{a} f(x)\,dx = 0$ for any integrable odd function, for the same reasons.

Exercise 11.2

1. (a) $\int_{-6}^{7} 2\,dx = [2x]_{-6}^{7} = 2(7) - 2(-6) = 26$

(b) $\int_{-1}^{5} (6x - 7)\,dx = \left[6\frac{x^2}{1+1} - 7x\right]_{-1}^{5} = \left[3(5)^2 - 7(5)\right] - \left[3(-1)^2 - 7(-1)\right] = 30$

(c) $\int_{1}^{2} (5 + 4x - 6x^2)\,dx = \left[5x + 4\frac{x^2}{1+1} - 6\frac{x^3}{2+1}\right]_{1}^{2}$

$= (10 + 8 - 16) - (5 + 2 - 2) = -3$

(d) $\int_{0}^{1} (t^2 + 6t - 1)\,dt = \left[\frac{1}{3}t^3 + 3t^2 - t\right]_{0}^{1} = \frac{1}{3} + 3 - 1 = \frac{7}{3}$

(e) $\int_{-1}^{2} (x^3 - x^2 + 4x)\,dx = \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 + 2x^2\right]_{-1}^{2} = \left(\frac{16}{4} - \frac{8}{3} + 8\right) - \left(\frac{1}{4} + \frac{1}{3} + 2\right) = \frac{27}{4}$

(f) $\int_{0}^{1} (x^{99} + 1)\,dx = \left[\frac{1}{100}x^{100} + x\right]_{0}^{1} = \frac{1}{100} + 1 = 1.01$

(g) $\int_{2}^{3} \left(\frac{1}{t^2}\right)dt = \left[\frac{t^{-1}}{-2+1}\right]_{2}^{3} = [-t^{-1}]_{2}^{3} = -\left(\frac{1}{3} - \frac{1}{2}\right) = \frac{1}{6}$

(h) $\int_{1}^{4} (x - \sqrt{x})\,dx = \left[\frac{x^2}{2} - \frac{2x^{\frac{3}{2}}}{3}\right]_{1}^{4} = \left(8 - \frac{16}{3}\right) - \left(\frac{1}{2} - \frac{2}{3}\right) = \frac{17}{6}$

(i) $\int_{0}^{1} \sqrt[4]{x^5}\,dx = \left[\frac{4x^{\frac{9}{4}}}{9}\right]_{0}^{1} = \frac{4}{9}$

(j) $\int_{1}^{8} \frac{2}{\sqrt[3]{x}}\,dx = \left[3x^{\frac{2}{3}}\right]_{1}^{8} = 3(4 - 1) = 9$

(k) $\int_{1}^{2} \frac{x^3 + x^2 + 1}{x^3}\,dx = \int_{1}^{2} \left(1 + \frac{1}{x} + \frac{1}{x^3}\right)dx = \left[x + \ln|x| - \frac{x^{-2}}{2}\right]_{1}^{2}$

$= \left(2 + \ln 2 - \frac{1}{8}\right) - \left(1 + \ln 1 - \frac{1}{2}\right) = \frac{11}{8} + \ln 2$

(l) $\int_{1}^{4} \left(\frac{\sqrt{x} + 1}{x}\right)dx = \int_{1}^{4} \left(\frac{1}{\sqrt{x}} + \frac{1}{x}\right) = [2\sqrt{x} + \ln x]_{1}^{4} = (4 + \ln 4) - (2 + \ln 1) = 2(1 + \ln 2)$

(m) $\int_{0}^{64} \sqrt{y}\left(1 + \sqrt[3]{y}\right)dy = \int_{0}^{64} \left(y^{\frac{1}{2}} + y^{\frac{5}{6}}\right)dy = \left[\frac{2y^{\frac{3}{2}}}{3} + \frac{6y^{\frac{11}{6}}}{11}\right]_{0}^{64} = \frac{1024}{3} + \frac{12288}{11} = \frac{48128}{33}$

(n) $\int_{0}^{\frac{\pi}{2}} (8x + \cos x)\,dx = [4x^2 + \sin x]_{0}^{\frac{\pi}{2}} = \left(\pi^2 + 1\right) - 0 = \pi^2 + 1$

(o) $\int_{0}^{\frac{\pi}{6}} (\sec x \tan x)\,dx = [\sec x]_{0}^{\frac{\pi}{6}} = \frac{2}{\sqrt{3}} - 1$

(p) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(3\sin\theta - \sec^2\theta)\,d\theta = \Big[-3\cos\theta - \tan\theta\Big]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left(-\tfrac{3}{2} - \sqrt{3}\right) - \left(-\tfrac{3}{\sqrt{2}} - 1\right)$

$= \tfrac{1}{2}(3\sqrt{2} - 1) - \sqrt{3}$

2. (a) $\displaystyle\int(x^5 - 2x^3 + 4)\,dx = \tfrac{1}{6}x^6 - \tfrac{1}{2}x^4 + 4x + C$

(b) $\displaystyle\int x^2\sqrt{x}\,dx = \int x^{\frac{5}{2}}\,dx = \frac{2x^{\frac{7}{2}}}{7} + C$

(c) $\displaystyle\int\left(t + \tfrac{2}{t}\right)dt = \tfrac{1}{2}t^2 + 2\ln|t| + C$

(d) $\displaystyle\int(1 + \sqrt{x})^2\,dx = \int(1 + 2\sqrt{x} + x)\,dx = x + \frac{4x^{\frac{3}{2}}}{3} + \frac{x^2}{2} + C$

(e) $\displaystyle\int\frac{x - 5}{\sqrt[4]{x}}\,dx = \int\left(x^{\frac{3}{4}} - 5x^{-\frac{1}{4}}\right)dx = \frac{4x^{\frac{7}{4}}}{7} - \frac{20x^{\frac{3}{4}}}{3} + C$

(f) $\displaystyle\int(\cos\theta + \sin\theta)\,d\theta = \sin\theta - \cos\theta + C$

(g) $\displaystyle\int(5x^4 - 2\csc x\cot x)\,dx = x^5 + 2\csc x + C$

(h) $\displaystyle\int(2\csc^2 x + 1)\,dx = x - 2\cot x + C$

3. (a) $\displaystyle\int_0^1 e^x\,dx = \Big[e^x\Big]_0^1 = e - 1$

(b) $\displaystyle\int_{-1}^1 2^x\,dx = \left[\frac{2^x}{\ln 2}\right]_{-1}^1 = \frac{1}{\ln 2}\left(2 - \tfrac{1}{2}\right) = \frac{3}{2\ln 2}$

(c) $\displaystyle\int_0^{\frac{1}{2}}\frac{1}{\sqrt{1 - x^2}}\,dx = \Big[\sin^{-1}x\Big]_0^{\frac{1}{2}} = \tfrac{\pi}{6} - 0 = \tfrac{\pi}{6}$

(d) $\displaystyle\int_1^{\sqrt{3}}\frac{12}{1 + x^2}\,dx = 12\Big[\tan^{-1}x\Big]_1^{\sqrt{3}} = 12\left(\tfrac{\pi}{3} - \tfrac{\pi}{4}\right) = \pi$

(e) $\displaystyle\int_{-1}^1\left(x + 1 + \frac{3}{1 + x^2}\right)dx = \left[\frac{x^2}{2} + x + 3\tan^{-1}x\right]_{-1}^1$

$\displaystyle= \left(\tfrac{1}{2} + 1 + \tfrac{3\pi}{4}\right) - \left(\tfrac{1}{2} - 1 - \tfrac{3\pi}{4}\right) = \frac{4 + 3\pi}{2}$

(f) $\displaystyle\int_{-\pi}^0(2e^x + \sin x)\,dx = \Big[2e^x - \cos x\Big]_{-\pi}^0 = (2 - 1) - (2e^{-\pi} + 1) = -2e^{-\pi}$

4. The Fundamental Theorem of Calculus (p. 501) holds true only for functions that are continuous on the closed interval over which you wish to integrate. In this case we require $f(x) = x^{-4}$ to be continuous on $[-2, 1]$. However, f is clearly not continuous at $x = 0$ (which is in $[-2, 1]$), and therefore we cannot find $\int_{-2}^1 x^{-4}\,dx$ using the Fundamental Theorem of Calculus as shown in the text.

Exercise 11.3

1. (a) Let $u = x^2$, then $du = 2x\,dx$.

(b) Let $u = \ln x$, then $du = \frac{dx}{x}$.

(c) Let $u = 5x$, then $du = 5\,dx$.

(d) Let $u = \sin x$, then $du = \cos x\,dx$.

2. (a) Let $u = 1 - x^2$, then $du = -2x\,dx$. So $-\frac{1}{2}du = x\,dx$, and,

$$\int x(1-x^2)^{10}dx = \int -\tfrac{1}{2}u^{10}du = -\tfrac{1}{22}u^{11} + C = -\tfrac{1}{22}(1-x^2)^{11} + C$$

(b) Let $u = 5x$, then $du = 5\,dx$. So, $\frac{1}{5}du = dx$, and,

$$\int e^{5x}dx = \int \tfrac{1}{5}e^u du = \tfrac{1}{5}e^u + C = \tfrac{1}{5}e^{5x} + C$$

(c) Let $u = x - 1$, then $du = dx$. So,

$$\int \sqrt{x-1}\,dx = \int \sqrt{u}\,du = \tfrac{2}{3}u^{\frac{3}{2}} + C = \tfrac{2}{3}(x-1)^{\frac{3}{2}} + C$$

(d) Let $u = x^2 + 2x - 6$, then $du = (2x+2)dx$. So $\frac{1}{2}du = (x+1)dx$, and,

$$\int \frac{x+1}{x^2+2x-6}dx = \int \frac{du}{2u} = \tfrac{1}{2}\ln|u| + C = \tfrac{1}{2}\ln|x^2+2x-6| + C$$

3. (a) Let $u = x^2 + 4$, then $du = 2x\,dx$. So $\frac{1}{2}du = x\,dx$, and,

$$\int x(x^2+4)^8 dx = \int \tfrac{1}{2}u^8 du = \tfrac{1}{18}u^9 + C = \tfrac{1}{18}(x^2+4)^9 + C$$

(b) Let $u = x^3 + 2$, then $du = 3x^2dx$. So $\frac{1}{3}du = x^2dx$, and,

$$\int x^2\sqrt{x^3+2}\,dx = \int \tfrac{1}{3}\sqrt{u}\,du = \tfrac{2}{9}u^{\frac{3}{2}} + C = \tfrac{2}{9}(x^3+2)^{\frac{3}{2}} + C$$

(c) Let $u = x + 6$, then $du = dx$. And,

$$\int (x+6)^{10}dx = \int u^{10}du = \tfrac{1}{11}u^{11} + C = \tfrac{1}{11}(x+6)^{11} + C$$

(d) Let $u = 3x - 1$, then $du = 3\,dx$. So $\frac{1}{3}du = dx$, and,

$$\int \frac{1}{(3x-1)^2}dx = \int \frac{du}{3u^2}du = -\frac{1}{3u} + C = -\frac{1}{3(3x-1)} + C$$

(e) Let $u = 3x$, then $du = 3\,dx$. So $\frac{1}{3}du = dx$, and,

$$\int \sec^2 3x\,dx = \int \tfrac{1}{3}\sec^2 u\,du = \tfrac{1}{3}\tan u + C = \tfrac{1}{3}\tan 3x + C$$

(f) Let $u = 1 + 2x^4$, then $du = 8x^3dx$. So $\frac{1}{8}du = x^3dx$, and,

$$\int (1+2x^4)x^3dx = \int \tfrac{1}{8}u\,du = \tfrac{1}{16}u^2 + C = \tfrac{1}{16}(1+2x^4) + C$$

(g) Let $u = \sin x$, then $du = \cos x\, dx$. So,

$$\int \sin^2 x \cos x\, dx = \int u^2\, du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\sin^3 x + C$$

(h) Let $u = \ln x$, then $du = \frac{dx}{x}$. So,

$$\int \frac{\sqrt{\ln x}}{x}\, dx = \int \sqrt{u}\, du = \frac{2u^{\frac{3}{2}}}{3} + C = \frac{2(\ln x)^{\frac{3}{2}}}{3} + C$$

(i) Let $u = t^3$, then $du = 3t^2 dt$. So $\frac{1}{3}du = t^2 dt$, and,

$$\int t^2 e^{t^3} dt = \int \tfrac{1}{3}e^u\, du = \tfrac{1}{3}e^u + C = \tfrac{1}{3}e^{t^3} + C$$

(j) Let $u = 1 - x$, then $du = -dx$. So $-du = dx$, and,

$$\int \frac{1}{1-x}\, dx = \int -\frac{du}{u} = -\ln|x| + C = -\ln|1-x| + C$$

(k) Let $u = x^3 - 2x + 1$, then $du = (3x^2 - 2)dx$. And,

$$\int \frac{3x^2 - 2}{(x^3 - 2x + 1)^3}\, dx = \int \frac{du}{u^3} du = -\frac{1}{2u^2} + C = -\frac{1}{2(x^3 - 2x + 1)^2} + C$$

(l) Let $u = \sqrt{x}$, then $du = \frac{dx}{2\sqrt{x}}$. So $2du = \frac{dx}{\sqrt{x}}$, and,

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = \int 2\sin u\, du = -2\cos u + C = -2\cos\sqrt{x} + C$$

(m) Let $u = 3 - x$, then $du = -dx$. So $-du = dx$, and,

$$\int e^{3-x}\, dx = \int -e^u\, du = -e^u + C = -e^{3-x} + C$$

(n) Let $u = \cos x$, then $du = -\sin x\, dx$. So $-du = \sin x\, dx$, and,

$$\int e^{\cos x}\sin x\, dx = \int -e^u\, du = -e^u + C = -e^{\cos x} + c$$

(o) Let $u = 1 + \tan x$, then $du = \sec^2 x\, dx$. So,

$$\int \sqrt{1 + \tan x}\, \sec^2 x\, dx = \int \sqrt{u}\, du = \tfrac{2}{3}u^{\frac{3}{2}} + C = \tfrac{2}{3}(1 + \tan x)^{\frac{3}{2}} + C$$

(p) Let $u = x^2$, then $du = 2x\, dx$. So $\frac{1}{2}du = x\, dx$, and,

$$\int x\sin(x^2)\, dx = \int \tfrac{1}{2}\sin u\, du = -\tfrac{1}{2}\cos u + C = -\tfrac{1}{2}\cos(x^2) + C$$

(q) Let $u = \cos x$, then $du = -\sin x\, dx$. So $-du = \sin x\, dx$, and,

$$\int \sin x \sin(\cos x)\, dx = \int -\sin u\, du = \cos u + C = \cos(\cos x) + C$$

(r) Let $u = \tan^{-1} x$, then $du = \frac{dx}{1 + x^2}$. So,

$$\int \frac{\tan^{-1} x}{1 + x^2}\, dx = \int u\, du = \tfrac{1}{2}u^2 + C = \tfrac{1}{2}(\tan^{-1} x)^2 + C$$

4. (a) Let $u = 2x + 1$, then $du = 2dx$.

When $x = 0$, $u = 1$. When $x = 1$, $u = 3$. So,

$$\int_0^1 e^{2x+1}\,dx = \int_1^3 \tfrac{1}{2}e^u\,du = \left[\tfrac{1}{2}e^u\right]_1^3 = \tfrac{1}{2}(e^3 - e)$$

(b) Let $u = 1 + 5x$, then $du = 5dx$.

When $x = 0$, $u = 1$. When $x = 2$, $u = 11$. So,

$$\int_0^2 \frac{1}{(1 + 5x)^4}\,dx = \int_1^{11} \frac{du}{5u^4} = \left[-\frac{1}{15u^3}\right]_1^{11} = -\tfrac{1}{15}\left(\tfrac{1}{1331} - 1\right) = \tfrac{266}{3993}$$

(c) Let $u = 4 - x^2$, then $du = -2x\,dx$.

When $x = 0$, $u = 4$. When $x = 2$, $u = 0$. So,

$$\int_0^2 x\sqrt{4 - x^2}\,dx = \int_4^0 -\tfrac{1}{2}\sqrt{u}\,du = \left[-\tfrac{1}{3}u^{\frac{3}{2}}\right]_4^0 = -\tfrac{1}{3}(0 - 8) = \tfrac{8}{3}$$

(d) Let $u = \pi t$, then $du = \pi\,dt$.

When $t = 0$, $u = 0$. When $t = 1$, $u = \pi$. So,

$$\int_0^1 \sin \pi t\,dt = \int_0^\pi \frac{\sin u}{\pi}\,du = \left[-\frac{\cos u}{\pi}\right]_0^\pi = -\tfrac{1}{\pi}(-1 - 1) = \tfrac{2}{\pi}$$

(e) Let $u = \sin\theta$, then $du = \cos\theta\,d\theta$.

When $\theta = \frac{\pi}{6}$, $u = \frac{1}{2}$. When $\theta = \frac{\pi}{2}$, $u = 1$. So,

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos\theta}{\sin^3\theta}\,d\theta = \int_{\frac{1}{2}}^1 \frac{du}{u^3} = \left[-\frac{1}{2u^2}\right]_{\frac{1}{2}}^1 = -\tfrac{1}{2}(1 - 4) = \tfrac{3}{2}$$

(f) Let $u = x^5 + 1$, then $du = 5x^4\,dx$.

When $x = 0$, $u = 1$. When $x = 1$, $u = 2$. So,

$$\int_0^1 x^4(x^5 + 1)^5\,dx = \int_1^2 \tfrac{1}{5}u^5\,du = \left[\tfrac{1}{30}u^6\right]_1^2 = \tfrac{1}{30}(64 - 1) = \tfrac{21}{10} = 2.1$$

(g) Let $u = 1 + \frac{1}{x}$, then $du = -\frac{dx}{x^2}\,dx$.

When $x = \frac{1}{2}$, $u = 3$. When $x = 1$, $u = 2$. So,

$$\int_{\frac{1}{2}}^1 \frac{\left(1 + \frac{1}{x}\right)^5}{x^2}\,dx = \int_3^2 -u^5\,du = \left[-\tfrac{1}{6}u^6\right]_3^2 = -\tfrac{1}{6}(64 - 729) = \tfrac{665}{6}$$

(h) Let $u = 3x^2 + 6x - 4$, then $du = 6(x + 1)\,dx$.

When $x = 1$, $u = 5$. When $x = 2$, $u = 20$. So,

$$\int_1^2 (x + 1)e^{3x^2+6x-4}\,dx = \int_5^{20} \tfrac{1}{6}e^u\,du = \left[\tfrac{1}{6}e^u\right]_5^{20} = \tfrac{1}{6}(e^{20} - e^5)$$

5. (a) Let $u = \cos x$, then $du = -\sin x\,dx$. So,

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \int -\frac{du}{u} = -\ln|u| + C = -\ln|\cos x| + C = \ln|\sec x| + C$$

(b) Let $u = \sin x$, then $du = \cos x\,dx$. So,

$$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|\sin x| + C$$

6. Since $f(x) = \sqrt{4x+1} \ge 0$ on $[0, 10]$, then $\int_0^{10} \sqrt{4x+1}\,dx$ can be interpreted as the area between the curve of f and the x-axis.

Let $u = 4x + 1$, then $du = 4\,dx$.

When $x = 0$, $u = 1$. When $x = 10$, $u = 41$. So,

$$A = \int_0^{10} \sqrt{4x+1}\,dx = \int_1^{41} \tfrac{1}{4}\sqrt{u}\,du = \left[\tfrac{1}{6}u^{\frac{3}{2}}\right]_1^{41} = \tfrac{1}{6}\left(41^{\frac{3}{2}} - 1\right)$$

7. Since $f(x) = \cos\left(\frac{x}{2}\right) \ge 0$ on $[0, \pi]$, then $\int_0^{\pi} \cos\left(\frac{x}{2}\right)dx$ can be interpreted as the area between the curve of f and the x-axis.

Let $u = \frac{x}{2}$, then $du = \frac{1}{2}dx$.

When $x = 0$, $u = 0$. When $x = \pi$, $u = \frac{\pi}{2}$. So,

$$A = \int_0^{\pi} \cos\left(\tfrac{x}{2}\right)dx = \int_0^{\frac{\pi}{2}} 2\cos u\,du = [2\sin u]_0^{\frac{\pi}{2}} = 2(1 - 0) = 2$$

8. Since $y = e^{2x} \ge y = e^{-x}$ on $[0, 1]$, the area bounded by these curves on the given interval is:

$$A = \int_0^1 \left(e^{2x} - e^{-x}\right)dx = \int_0^1 e^{2x}\,dx - \int_0^1 e^{-x}\,dx$$

For $\int_0^1 e^{2x}\,dx$, Let $u = 2x$, then $du = 2\,dx$.

When $x = 0$, $u = 0$. When $x = 1$, $u = 2$. So,

$$\int_0^1 e^{2x}\,dx = \int_0^2 \tfrac{1}{2}e^u\,du = \left[\tfrac{1}{2}e^u\right]_0^2 = \tfrac{1}{2}(e^2 - 1)$$

For $\int_0^1 e^{-x}\,dx$, let $u = -x$, then $du = -dx$.

When $x = 0$, $u = 0$. When $x = 1$, $u = -1$. So,

$$\int_0^1 e^{-x}\,dx = \int_0^{-1} -e^u\,du = \left[-e^u\right]_0^{-1} = 1 - e^{-1}$$

Therefore, $A = \frac{1}{2}(e^2 - 1) - (1 - e^{-1}) = \frac{1}{2}(e^2 - 3) + e^{-1}$

9. (a) Let $u = \sqrt{x} + 1$, then $du = \dfrac{dx}{2\sqrt{x}}$. So,

$$\int \frac{1}{x + \sqrt{x}}dx = \int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx = \int \frac{2du}{u} = 2\ln|u| + C = 2\ln(\sqrt{x}+1) + C$$

[OR: Let $u = \sqrt{x}$, then $u^2 = x$, so $dx = 2u\,du$ and

$$\int \frac{1}{x+\sqrt{x}}dx = \int \frac{2u\,du}{u^2+u} = 2\int \frac{1}{u+1}du = 2\ln|u+1| + C = 2\ln(\sqrt{x}+1) + C$$]

(b) Let $u = x + 2$, then $x + 1 = u - 1$, and $du = dx$. So,

$$\int \frac{x+1}{x+2}dx = \int \frac{u-1}{u}du = \int \left(1 - \frac{1}{u}\right)du = u - \ln|u| + C = x + 2 - \ln|x+2| + C$$